

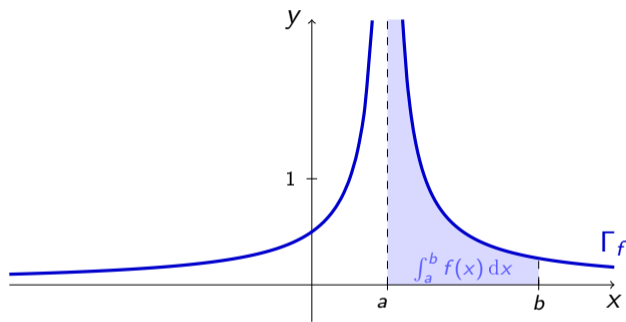


5.7.2. Nepravi integrali neograničenih funkcija

8. 1. 2020.

Definicija 1(a)

Neka su $a, b \in \mathbb{R}$ takvi da je $a < b$.

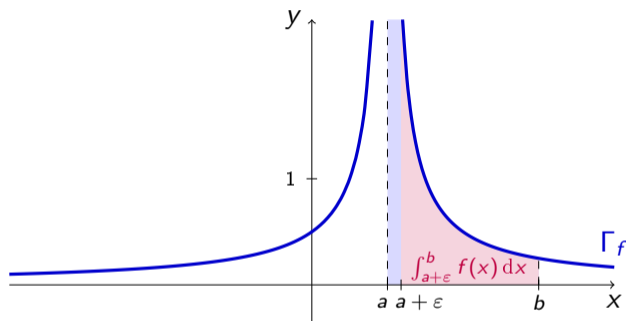


Ako je pravac $x = a$ vertikalna asimptota funkcije $f : D \subseteq \mathbb{R} \rightarrow \mathbb{R}$, tada definiramo

$$\int_a^b f(x) dx :=$$

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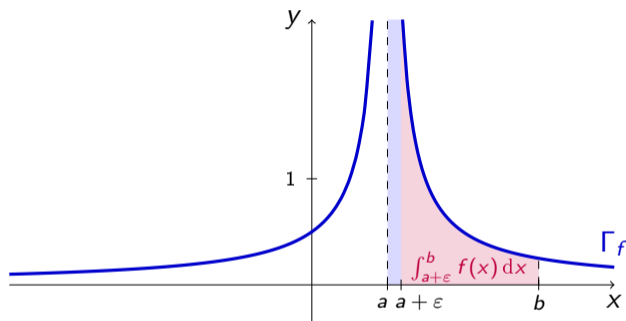


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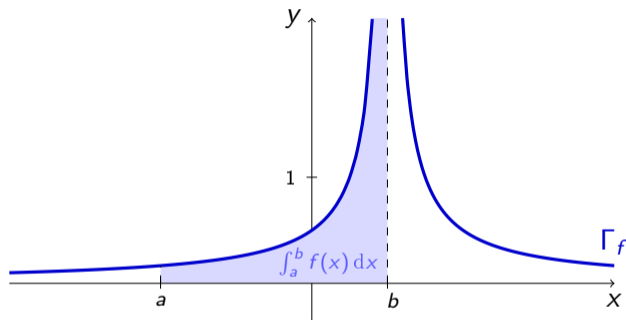
Ako je pravac $x = a$ vertikalna asimptota funkcije $f : D \subseteq \mathbb{R} \rightarrow \mathbb{R}$, tada definiramo

$$\int_a^b f(x) dx := \lim_{\epsilon \rightarrow 0^+} \int_{a+\epsilon}^b f(x) dx,$$

kad god je desna strana ove formule definirana.

Definicija 1(b)

Neka su $a, b \in \mathbb{R}$ takvi da je $a < b$.

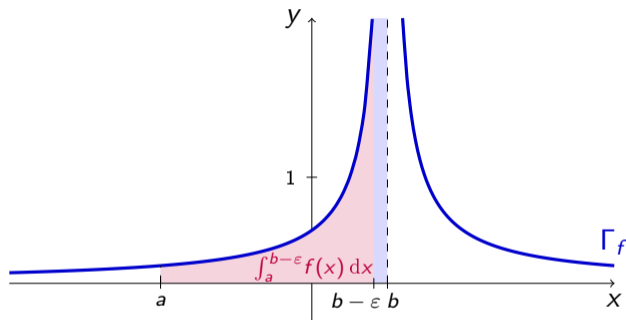


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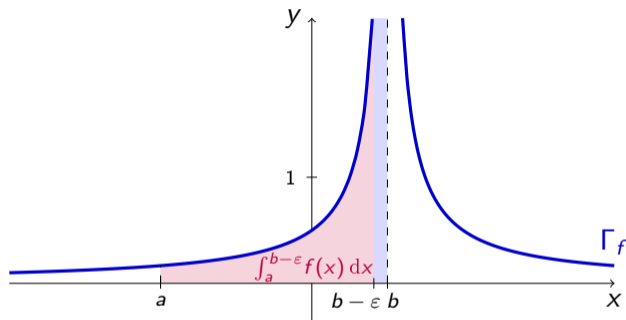


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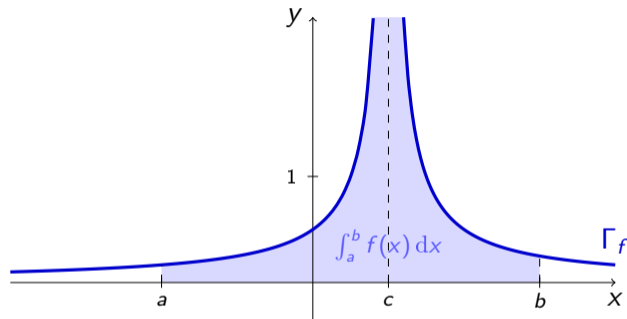
Ako je pravac $x = b$ vertikalna asimptota funkcije $f : D \subseteq \mathbb{R} \rightarrow \mathbb{R}$, tada definiramo

$$\int_a^b f(x) dx := \lim_{\epsilon \rightarrow 0^+} \int_a^{b-\epsilon} f(x) dx,$$

kad god je desna strana ove formule definirana.

Definicija 1(c)

Neka su $a, b, c \in \mathbb{R}$ takvi da je $a < c < b$.

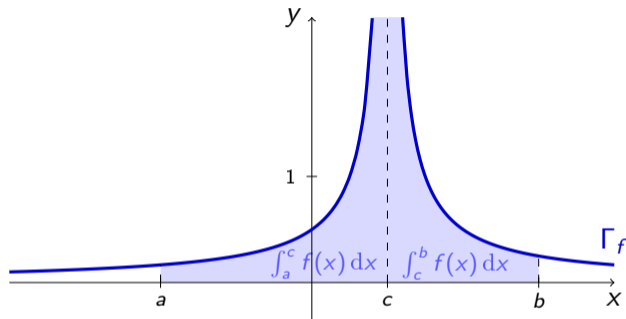


Ako je pravac $x = c$ vertikalna asimptota funkcije $f : D \subseteq \mathbb{R} \rightarrow \mathbb{R}$, tada definiramo

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Neka su $a, b, c \in \mathbb{R}$ takvi da je $a < c < b$.

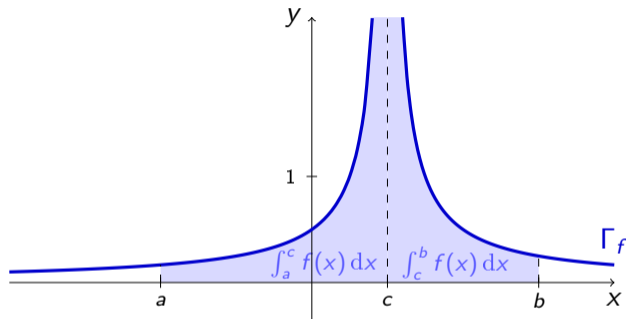


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$$\int_a^b f(x) dx :=$$

Definicija 1(c)

Neka su $a, b, c \in \mathbb{R}$ takvi da je $a < c < b$.



Ako je pravac $x = c$ vertikalna asimptota funkcije $f : D \subseteq \mathbb{R} \rightarrow \mathbb{R}$, tada definiramo

$$\int_a^b f(x) dx := \underbrace{\int_a^c f(x) dx}_{\text{Definicija 1(b)}} + \underbrace{\int_c^b f(x) dx}_{\text{Definicija 1(a)}}$$

kad god je desna strana ove formule definirana.

Zadatak 54(a)

Izračunajte integral $\int_0^1 \frac{dx}{\sqrt{x}}$.

$$\int dx = x + C$$

$$\int x^a dx = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{dx}{x} = \ln|x| + C$$

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$$\begin{aligned}\int_0^1 \frac{dx}{\sqrt{x}} &= \lim_{\varepsilon \rightarrow 0^+} \int_{\varepsilon}^1 \frac{dx}{\sqrt{x}} \\ &= \lim_{\varepsilon \rightarrow 0^+} 2\sqrt{x} \Big|_{\varepsilon}^1\end{aligned}$$

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Izračunajte integral $\int_{-1}^1 \frac{dx}{x^2}$.

Rješenje. Kako je pravac $x = 0$ vertikalna asimptota podintegralne funkcije $\frac{1}{x^2}$ i vrijedi $0 \in [-1, 1]$, imamo

$$\int_{-1}^1 \frac{dx}{x^2} = \int_{-1}^0 \frac{dx}{x^2} + \int_0^1 \frac{dx}{x^2}$$

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$$\int \frac{dx}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

Zadatak 54(c)

Izračunajte integral $\int_{-1}^1 \frac{dx}{x^2}$.

Rješenje. Kako je pravac $x = 0$ vertikalna asimptota podintegralne funkcije $\frac{1}{x^2}$ i vrijedi $0 \in [-1, 1]$, imamo

$$\begin{aligned}\int_{-1}^1 \frac{dx}{x^2} &= \int_{-1}^0 \frac{dx}{x^2} + \int_0^1 \frac{dx}{x^2} \\ &= \lim_{\varepsilon \rightarrow 0^+} \int_{-1}^{-\varepsilon} \frac{dx}{x^2} + \lim_{\varepsilon \rightarrow 0^+} \int_{\varepsilon}^1 \frac{dx}{x^2} \\ &= \lim_{\varepsilon \rightarrow 0^+} \left(-\frac{1}{x} \right) \Big|_{-1}^{-\varepsilon} + \lim_{\varepsilon \rightarrow 0^+} \left(-\frac{1}{x} \right) \Big|_{\varepsilon}^1\end{aligned}$$

$$\int dx = x + C$$
$$\int x^a dx = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{dx}{x} = \ln|x| + C$$

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Zadatak 54(c)

Izračunajte integral $\int_{-1}^1 \frac{dx}{x^2}$.

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$$\begin{aligned}\int dx &= x + C \\ \int x^a dx &= \frac{x^{a+1}}{a+1} + C \quad (a \neq -1) \\ \int \frac{dx}{x} &= \ln|x| + C \\ \int e^x dx &= e^x + C \\ \int a^x dx &= \frac{a^x}{\ln a} + C \quad (a > 0, a \neq 1) \\ \int \cos x dx &= \sin x + C \\ \int \sin x dx &= -\cos x + C \\ \int \frac{dx}{\cos^2 x} &= \operatorname{tg} x + C \\ \int \frac{dx}{\sin^2 x} &= -\operatorname{ctg} x + C \\ \int \frac{dx}{\sqrt{1-x^2}} &= \arcsin x + C \\ \int \frac{dx}{1+x^2} &= \operatorname{arctg} x + C \\ \int \frac{dx}{a^2+x^2} &= \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)\end{aligned}$$

Zadatak 54(c)

Izračunajte integral $\int_{-1}^1 \frac{dx}{x^2}$.

Rješenje. Kako je pravac $x = 0$ vertikalna asimptota podintegralne funkcije $\frac{1}{x^2}$ i vrijedi $0 \in [-1, 1]$, imamo

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$$\begin{aligned}\int dx &= x + C \\ \int x^a dx &= \frac{x^{a+1}}{a+1} + C \quad (a \neq -1) \\ \int \frac{dx}{x} &= \ln|x| + C \\ \int e^x dx &= e^x + C \\ \int a^x dx &= \frac{a^x}{\ln a} + C \quad (a > 0, a \neq 1) \\ \int \cos x dx &= \sin x + C \\ \int \sin x dx &= -\cos x + C \\ \int \frac{dx}{\cos^2 x} &= \operatorname{tg} x + C \\ \int \frac{dx}{\sin^2 x} &= -\operatorname{ctg} x + C \\ \int \frac{dx}{\sqrt{1-x^2}} &= \arcsin x + C \\ \int \frac{dx}{1+x^2} &= \operatorname{arctg} x + C \\ \int \frac{dx}{a^2+x^2} &= \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)\end{aligned}$$

Zadatak 54(c)

Izračunajte integral $\int_{-1}^1 \frac{dx}{x^2}$.

Rješenje. Kako je pravac $x = 0$ vertikalna asimptota podintegralne funkcije $\frac{1}{x^2}$ i vrijedi $0 \in [-1, 1]$, imamo

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$$\begin{aligned}\int dx &= x + C \\ \int x^a dx &= \frac{x^{a+1}}{a+1} + C \quad (a \neq -1) \\ \int \frac{dx}{x} &= \ln|x| + C \\ \int e^x dx &= e^x + C \\ \int a^x dx &= \frac{a^x}{\ln a} + C \quad (a > 0, a \neq 1) \\ \int \cos x dx &= \sin x + C \\ \int \sin x dx &= -\cos x + C \\ \int \frac{dx}{\cos^2 x} &= \operatorname{tg} x + C \\ \int \frac{dx}{\sin^2 x} &= -\operatorname{ctg} x + C \\ \int \frac{dx}{\sqrt{1-x^2}} &= \arcsin x + C \\ \int \frac{dx}{1+x^2} &= \operatorname{arctg} x + C \\ \int \frac{dx}{a^2+x^2} &= \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)\end{aligned}$$

Zadatak 54(c)

Izračunajte integral $\int_{-1}^1 \frac{dx}{x^2}$.

$$\int dx = x + C$$

$$\int x^a dx = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{dx}{x} = \ln|x| + C$$

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Zadatak 54(c)

Izračunajte integral $\int_{-1}^1 \frac{dx}{x^2}$.

Krivo rješenje.

$$\int dx = x + C$$

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Zadatak 54(c)

Izračunajte integral $\int_{-1}^1 \frac{dx}{x^2}$.

Krivo rješenje. Imamo $\int_{-1}^1 \frac{dx}{x^2} = -\frac{1}{x} \Big|_{-1}^1$

$$\int dx = x + C$$

$$\int x^a dx = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{dx}{x} = \ln|x| + C$$

$$\int e^x dx = e^x + C$$

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Zadatak 54(c)

Izračunajte integral $\int_{-1}^1 \frac{dx}{x^2}$.

Krivo rješenje. Imamo
$$\int_{-1}^1 \frac{dx}{x^2} = -\frac{1}{x} \Big|_{-1}^1$$
$$= -\frac{1}{1} - \left(-\frac{1}{-1}\right)$$

$$\int dx = x + C$$

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Zadatak 54(c)

Izračunajte integral $\int_{-1}^1 \frac{dx}{x^2}$.

Krivo rješenje. Imamo
$$\begin{aligned}\int_{-1}^1 \frac{dx}{x^2} &= -\frac{1}{x} \Big|_{-1}^1 \\ &= -\frac{1}{1} - \left(-\frac{1}{-1}\right) \\ &= -2.\end{aligned}$$

$$\int dx = x + C$$
$$\int x^a dx = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{dx}{x} = \ln|x| + C$$

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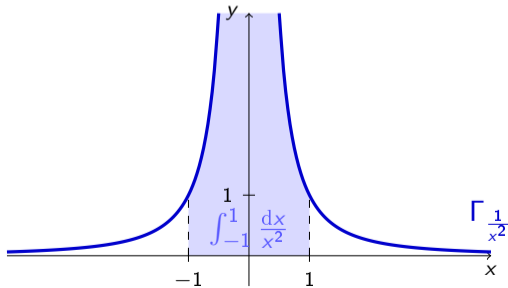
$$\int \frac{dx}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

Zadatak 54(c)

Izračunajte integral $\int_{-1}^1 \frac{dx}{x^2}$.

Krivo rješenje. Imamo
$$\int_{-1}^1 \frac{dx}{x^2} = -\frac{1}{x} \Big|_{-1}^1$$
$$= -\frac{1}{1} - \left(-\frac{1}{-1}\right)$$
$$= -2.$$



$$\int dx = x + C$$
$$\int x^a dx = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

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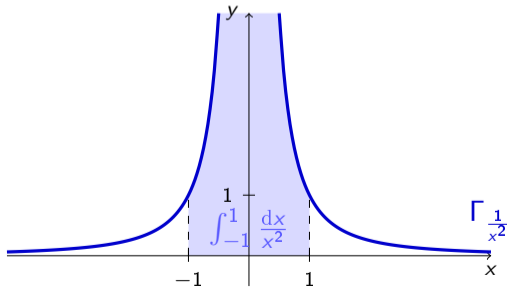
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$$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

Zadatak 54(c)

Izračunajte integral $\int_{-1}^1 \frac{dx}{x^2}$.

Krivo rješenje. Imamo $\int_{-1}^1 \frac{dx}{x^2} \neq -\frac{1}{x} \Big|_{-1}^1$
 $= -\frac{1}{1} - \left(-\frac{1}{-1}\right)$
 $= -2.$



$$\int dx = x + C$$
$$\int x^a dx = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{dx}{x} = \ln|x| + C$$

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